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Significance of P-Δ Effects on Seismic Response of R/C Structures

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ABSTRACT

Analytical investigation was carried out to establish the significance of P-Δ effects on seismic response of reinforced concrete buildings. A 10-story frame building was designed and analyzed for two distinctly different seismic regions in Canada. Both static inelastic (push-over) and dynamic inelastic analyses were conducted to assess the importance of secondary moments and related deformations on building response. The results indicate that P-Δ effects can be quite significant, especially if the degree of inelasticity is high, with increased inelastic deformations. Buildings that showed essentially elastic behavior did not exhibit significant differences in response due to P-Δ effects. Building response in relatively mild seismic conditions of eastern Canada was in this category, and did not show sizeable P-Δ effects to warrant a second order analysis. Push over analysis, however, produced lateral force and ductility capacities that were substantially lower than those indicated by an equivalent analysis without the consideration of P-Δ effects.

INTRODUCTION

Reinforced concrete structures subjected to seismic excitations may undergo significant lateral deformations, especially in the inelastic range. Vertical elements of these structures may be subjected to high axial compression caused by gravity and/or seismic induced inertia loads. First story columns of multistory buildings form a good example of a structural element that may experience a combination of high lateral deformations and axial compression. This combination creates sizeable secondary stresses (P-Δ effects), and may lead to structural failures earlier than the anticipated strength and deformation levels. Often, vertical elements like columns are responsible from strength and stability of the entire structure, and become critical during seismic response. Therefore, the consideration of secondary effects, such as the P-Δ effects, becomes an essential component of inelastic analysis of structures for improved accuracy of results.

A 10-story reinforced concrete building was designed for locations in eastern and western Canada to investigate the significance of P-Δ effect. The building was analyzed inelastically under static and dynamic conditions, using computer software DRAIN-RC (Saatcioglu et al. 1999). The software incorporates appropriate options for static inelastic (push-over) and dynamic inelastic analyses, with P-Δ effects included in both cases. Appropriate hysteretic models for reinforced concrete response are also incorporated, accounting for deformation components caused by flexure, shear, and anchorage slip. The details of the analyses and the results are presented and discussed in the paper.

STRUCTURAL ANALYSIS

Computer Software

Computer software DRAIN-RC (Saatcioglu et al. 1999), developed at the University of Ottawa by modifying the general purpose software DRAIN-2D (Kannan and Powell 1973), was used to conduct static and dynamic inelastic analyses. DRAIN-RC was specifically developed for analysis of reinforced concrete structures, with appropriate hysteretic models. The models, illustrated in Figure 1, account for flexure, shear, anchorage slip, M-P interaction, and infill panels. The structure is first idealized for analysis as a planar assemblage of discrete elements. Each node has up to three degrees of freedom, which can be restrained at supports or can be combined with other nodal degrees of freedom. Each reinforced concrete element is modeled as an elastic line element with three springs at each end to account for inelasticity due to flexure, shear, and anchorage slip. The hysterstic models depicted in Figure 1 are assigned to these springs, and are activated in the inelastic range of deformations, while the elastic beam with appropriate stiffnesses account for the elastic portion of total deformation. The global stiffness matrix is formed by assembling individual element stiffnesses, which are determined by the direct stiffness method.

Static inelastic analysis (Push-over analysis) is an option in the software. In this option, the nodal loads can be applied incrementally in several load steps. The global stiffness matrix is updated for each load step, and if yielding is encountered unbalanced forces are computed and added to the load vector in the following load step. Some nodal loads, like gravity loads, can be applied at their full magnitudes, while others can be specified as incremental loads, increasing up to failure.

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Dynamic response is computed following the step-by-step integration method, with constant acceleration assumed during each time step. Structural mass can be assigned to the nodes. Damping is expressed as a combination of mass and stiffness dependant matrices. Horizontal and vertical accelerations of ground motion can be input at selected time intervals.

The P-Δ effect is introduced through the modification of global stiffness matrix, with due considerations given to the change in flexural rigidity terms because of changes in the geometric stiffness matrix. This implies that the P-Δ effect is accounted for by reducing the stiffness of the system so that an additional displacement due to this effect is obtained. The geometric stiffness matrix, incorporated in the computer software, is shown below:

$$K_G = \frac{N}{30L}\begin{vmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 36 & 3L & 0 & -36 & 3L \\ 0 & 3L & 4L^2 & 0 & -3L & -L^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -36 & -3L & 0 & 36 & -3L \\ 0 & 3L & -L^2 & 0 & -3L & 4L^2 \end{vmatrix} \qquad (1)$$

where, L is the length of an element and N is the axial load on the element. The geometric stiffness K_G is computed for each element, as indicated in Eq. 1, and subtracted from the flexural stiffness matrix at the beginning of analysis. If the axial force on an element changes, the geometric stiffness is calculated at each time step during dynamic analysis, and at each load step for push-over analysis.

Structure Selected for Analysis

A 10-story frame building was designed for Vancouver and Ottawa, representing western and eastern Canadian seismic regions. The building was designed following the seismic provisions of the National Building Code of Canada (NBCC 1995). Figure 2 illustrates a typical floor plan, as well as the elevation view. The interior frames in the short direction had 500 mm, and 400 mm square sections for interior and edge columns, respectively. The exterior frames in the same direction had 450 mm and 350 mm square sections for edge and corner columns, respectively. All the beams were 300 mm wide and 450 mm deep, except at the roof level, where the beam depth was reduced to 400 mm.

Static Inelastic (Push-Over) Analysis

Push-over analysis of the building designed for Vancouver was conducted by applying incrementally increasing equivalent seismic forces as lateral loads, while keeping gravity loads on the structure. The distribution of the lateral load was the same as that recommended in the National Building Code of Canada (NBCC 1995). The gravity loads contributed to structural response, including the secondary moments caused by P-Δ. Inelasticity in static analysis was considered through the same inelastic springs used for dynamic analysis. The same hysteretic models for flexure, shear and anchorage slip were used, except this time only the primary curves under monotonic loading were utilized. The axial force-flexure interaction during response was not included, as this effect was found to be insignificant for the frame structure considered in analysis.

The structure was analyzed twice; with and without the P-Δ effects. Lateral force-lateral drift relationships for both cases are compared in Figure 3 for roof, as well as 5th floor level where the inter-story drift was the highest. The results indicate that the P-Δ effect was very significant. Without this effect, the building continued resisting increased lateral loads beyond realistic values of drift and ductility ratio. The maximum inter-storey drift was in excess of 4.0 % and the maximum ductility ratio computed in beams was in excess of 15. The maximum load resistance was approximately equal to 200 % of the design base shear when the analysis was terminated. The same building showed a more realistic behavior when secondary deformations due to P-Δ were considered. The building experienced a stability failure at a maximum inter-story drift of about 1 %. The over-strength ratio, relative to the design base shear, was approximately 1.2 when the structure collapsed. At this load stage, the maximum beam ductility ratio was 3.4, while the columns remained elastic. The comparison given in Figure 3 indicates that P-Δ effects play an important role on inelastic static behavior of buildings. Ignoring these effects can lead to erroneous results, giving very high force and deformation capacities.

Ground Motion Records

An inventory of earthquake records were considered for eastern and western Canada for dynamic response history analysis.

The records consisted of actual recorded earthquake motions and artificially generated motions. The actual earthquake records of significance in eastern Canada are limited in numbers. Two such records were selected for this region, consisting of 1988 Saguenay, Quebec Earthquake and 1982 Miramichi, New Brunswick Earthquake. The artificial earthquake motions were generated by Atkinson and Beresnew (1998), and consisted of four records for each site. There was no actual earthquake record available for western Canada. Therefore, previous earthquake motions recorded in western U.S.A. were used, although it may be argued that the seismological activities of western U.S.A. and western Canada could be different. A total of seven records were considered for the west, consisting of 1940 El-Centro Earthquake, 1952 Taft Record, 1971 San Fernando Earthquake, and four different records of the 1994 Northridge Earthquake. In addition, four artificial records generated by Atkinson and Beresnev (1998) were considered. All the records were normalized to give Peak Ground Accelerations indicated in NBCC (1995) for 10% probability of exceedence in 50 years. Among the artificial records considered, Long Event # 2 for the east and west produced highest response. Among the actual records, the 1940 El-Centro record and the 1988 Saguenay record gave the highest response for the west and east, respectively.

Dynamic Inelastic Response History Analysis

The 10-story buildings designed for western and eastern Canada were analyzed under critical ground motions, with and without P-Δ effects. Maximum drift and inter-story drift are plotted in Figures 4 and 5. The results indicate a significant change in response with the consideration of P-Δ effects, especially in the building designed for Vancouver. The building for Ottawa did not show any appreciable difference. This was explained by the degree of inelasticity and hence the amount of inelastic deformations experienced during response. The building designed for Ottawa, when subjected to eastern earthquake records, did not develop any appreciable inelasticity. Hence, the P-Δ effect was small. In contrast, the building for Vancouver, subjected to higher intensity western earthquakes, developed increased inelasticity and showed a significant change in response. This point was further investigated under increased intensity of ground motions. The building was re-analyzed under Long Event No.2 (artificial record) and 1940 El-Centro record with accelerations scaled by 150 % and 200 % to give peak ground accelerations of 34.5% g (gravitational acceleration) and 46% g, respectively. These records resulted in increased inelasticity. When the building was subjected to the scaled 1940 El Centro record with a peak ground acceleration of 34.5 % g, the maximum beam ductility ratios increased to approximately 8 and 12 when the P-Δ effect was ignored and considered, respectively. Maximum column ductility ratios for the same two cases were approximately 1.6 and 2.1, respectively. Figure 6 indicates an increase of 35 % in drift response and 40 % in maximum inter-storey drift. When the building was subjected to 1940 El-Centro record, scaled to give a peak ground acceleration of 45 % g, the building collapsed when the P-Δ effect was considered in analysis. The same building, with the same ground motion survived the earthquake with a maximum drift of 5 %, when the P-Δ effect was ignored. These analyses provide sufficient data to conclude that P-Δ effects play a crucial role on building response, and that they can not be ignored in response history analysis for earthquake effects, especially if the level of inelasticity is high.

SUMMARY AND CONCLUSIONS

A 10-story reinforced concrete frame structure was designed and analyzed for two locations in Canada; Vancouver, and Ottawa, representing western and eastern Canada. The building was first analyzed under incrementally increasing equivalent static loads. It was then subjected to earthquake ground motions, representative of eastern and western Canadian seismicity. The results indicate that secondary moments and deformations caused by P-Δ effects can be very significant in static inelastic (push-over) analysis. Analysis without the proper consideration of P-Δ effect can lead to serious overestimation of load and deformation capacities. Dynamic response can also be affected by P-Δ effects. When the deformations are small, as in the case of elastically behaving structures, the P-Δ effect may not be important, and may be ignored without significantly affecting the accuracy of results. However, when significant yielding is expected during response, the P-Δ effects gain importance, and may result in completely different response. The response history analyses indicate that drift demands can be increased by up to 40 %, when the column ductility demand is approximately 2.0. When the ductility demand is increased the increase in drift demands due to P-Δ effects can be higher, resulting in a stability failure, a failure mode which would otherwise not be captured if the P-Δ effects were ignored in analysis. Therefore, it is important to include P-Δ effects in static and dynamic inelastic analyses of reinforced concrete structures, especially when the anticipated level of inelasticity is high.

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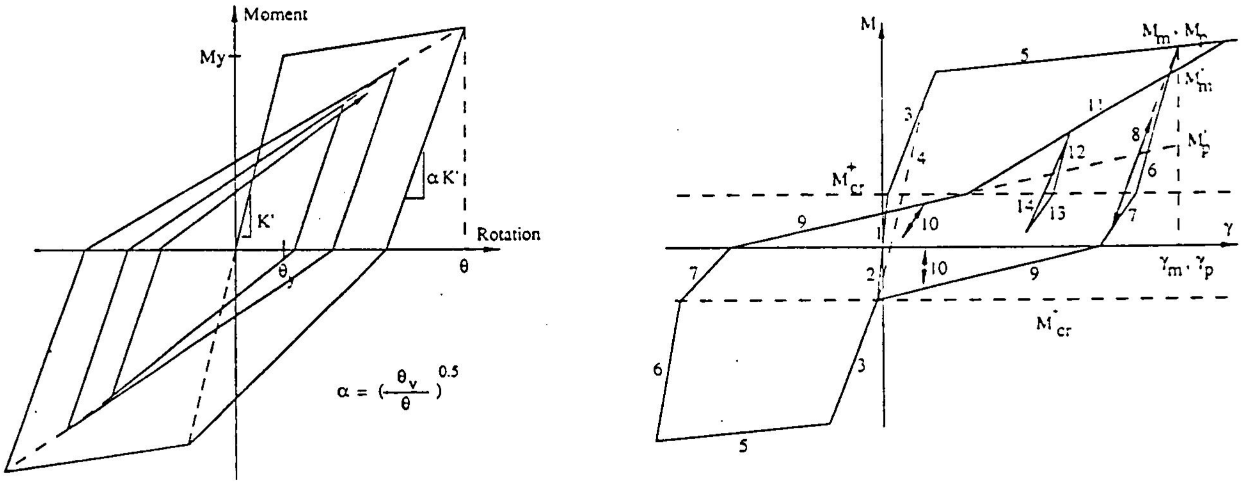
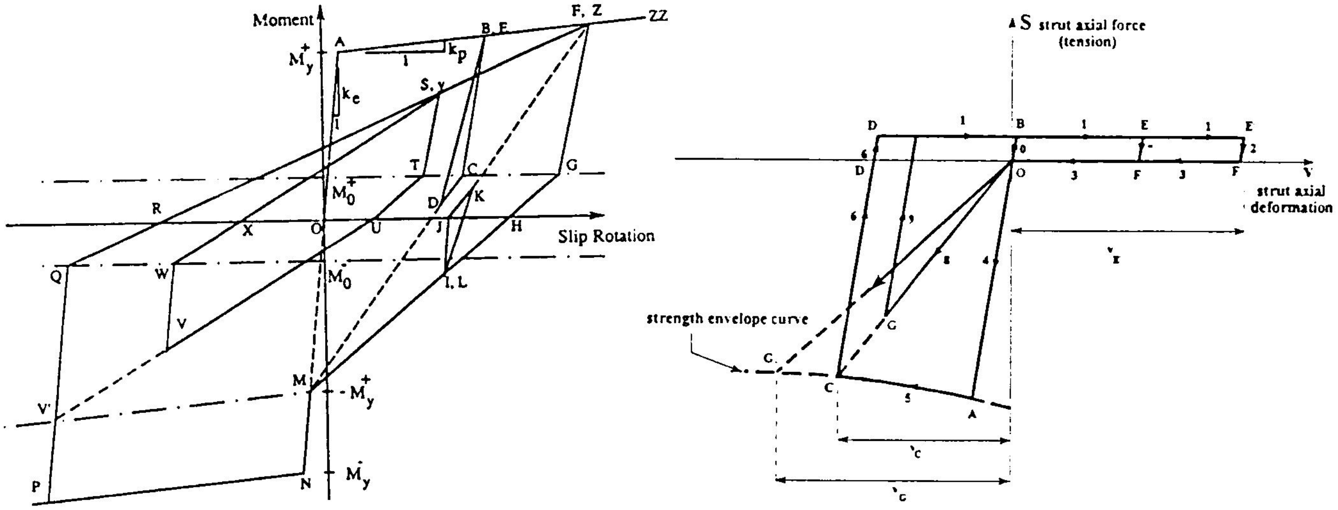


a) Model for flexure (Takeda et al. 1970)

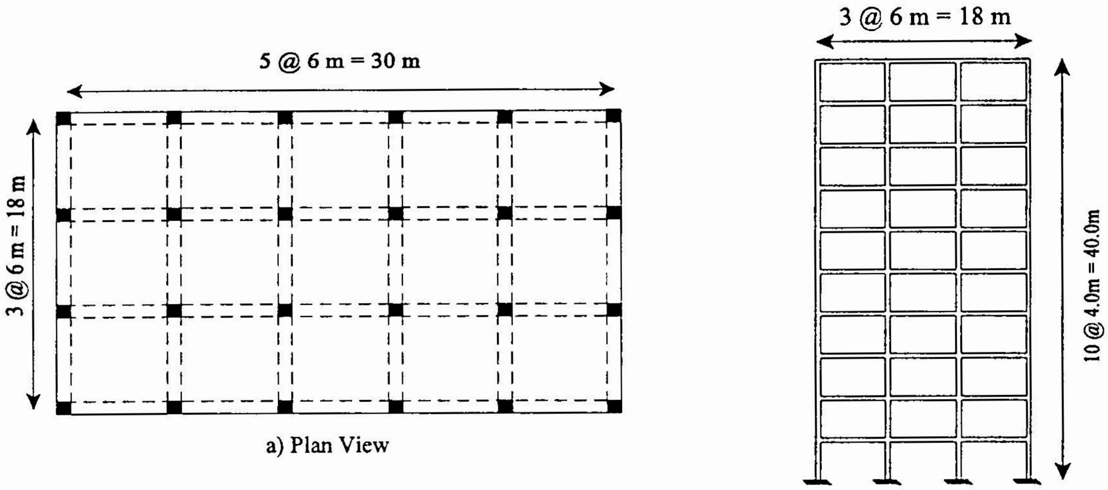
b) Model for shear (Ozcebe and Saatcioglu 1989)



c) Model for anchorage slip (Saatcioglu et al. 1992)

d) Model for infill walls (Klingner and Bertero 1978)

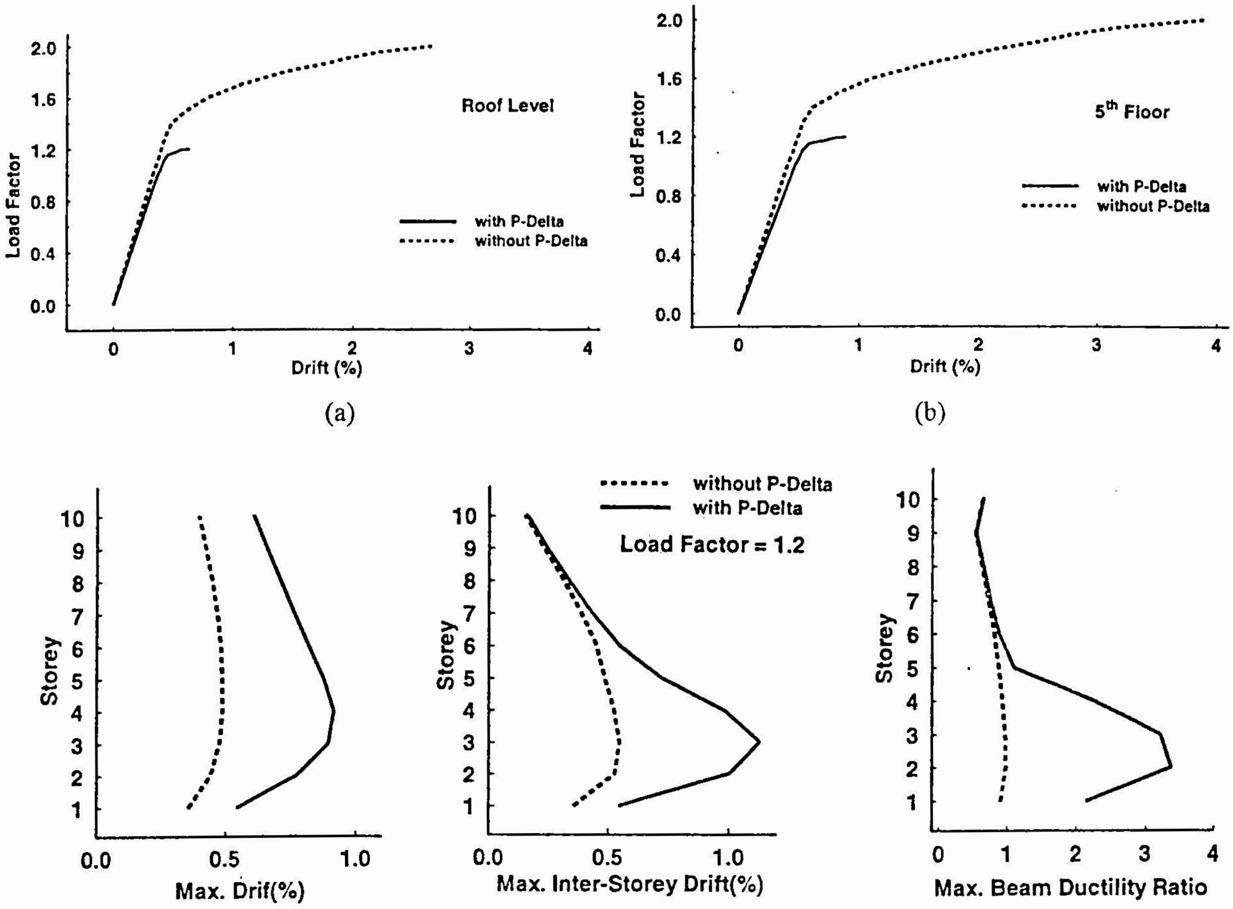
Figure 1 Hysteretic models



a) Plan View

b) Elevation

Figure 2 Details of the 10-Story building designed and analyzed



(a)

(b)

(c)

(d)

(e)

Figure 3 Drift response under incrementally increasing static lateral force (push-over analysis).

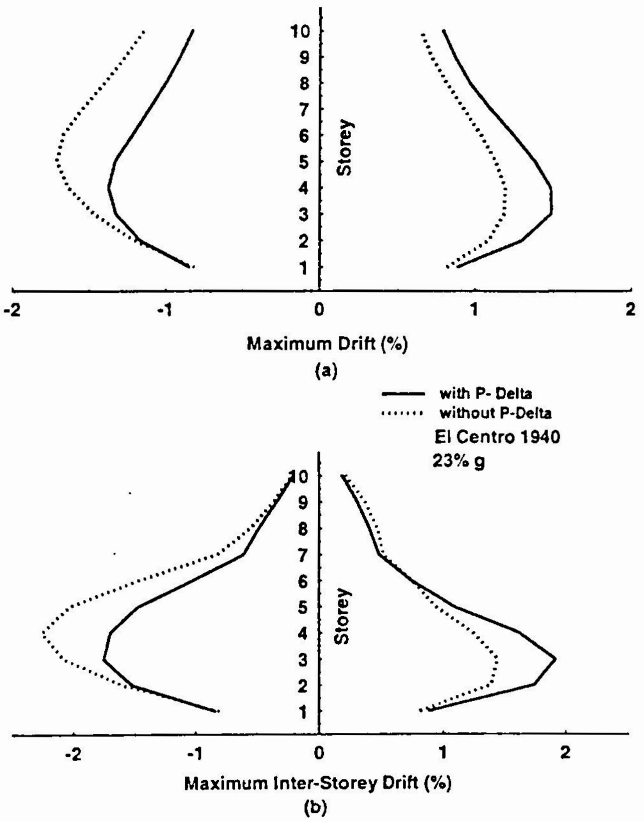


Figure 4 Drift response for western Canada

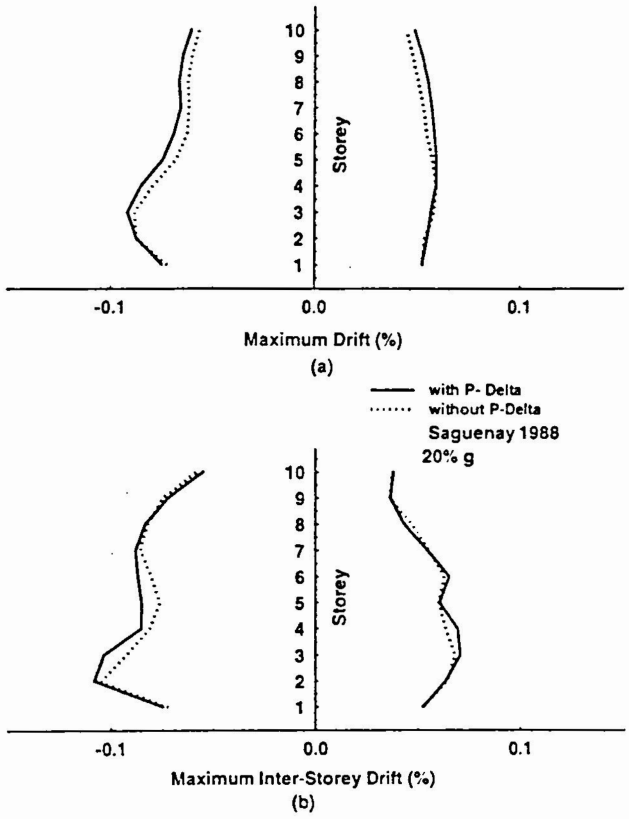


Figure 5 Drift response for eastern Canada

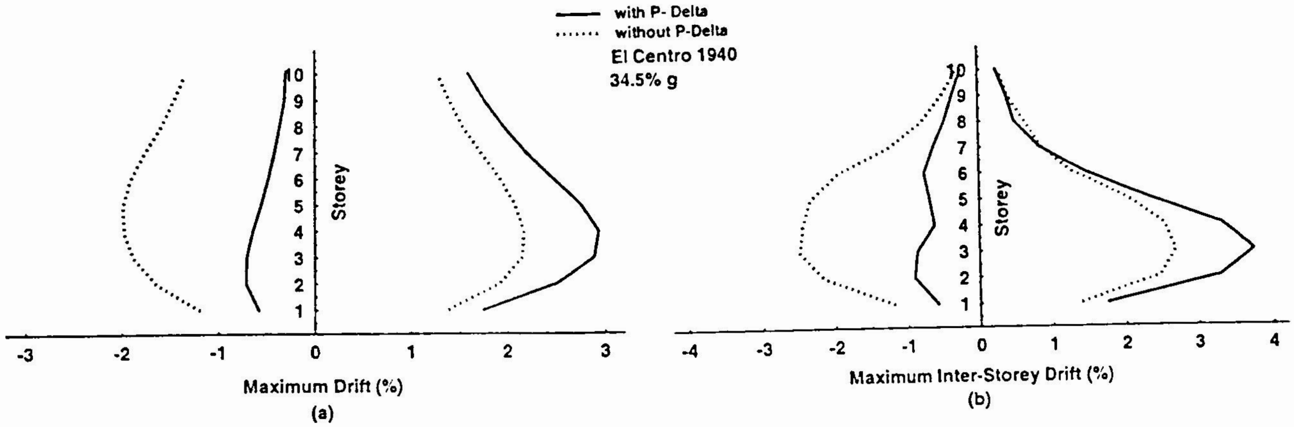


Figure 6 Drift response under increased intensity of ground motion